Check $T: P_2 \to P_2$ $\quad T p(x) = p(x+1)$ is a linear Transform.

$T(cp(x)) = cp(x+1) = c\,Tp(x)$

$T(P_1 + P_2)(x) = (P_1 + P_2)(x+1) = P_1(x+1) + P_2(x+1)$

Two function add together means function value add together

$= TP_1(x) + TP_2(x)$

How to think about this Question

$p(x) = ax^2 + bx + c$

$\Rightarrow p(x+1) = a(x+1)^2 + b(x+1) + c$

$= ax^2 + 2ax + a + bx + b + c$

$= ax^2 + (2a+b)x + a+b+c$

$Tp(x) = p(x+1)$ is Transforming $\begin{bmatrix} a \\ b \\ c \end{bmatrix} \longrightarrow \begin{bmatrix} a \\ 2a+b \\ a+b+c \end{bmatrix}$

$\Rightarrow$ Matrix: $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$