$\lambda = a + bi$, Conjugate complex number $\lambda^* = a - bi$

A is symmetric and real

real number means $\lambda = \lambda^*$

$$\vec{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \qquad \vec{x}^* = \begin{pmatrix} x_1^* \\ \vdots \\ x_n^* \end{pmatrix}$$

$\rightarrow$ Conjugate every element.

$(\vec{x}^*)^T \vec{x}$ is real number ! $(a+bi) \cdot (a-bi) = a^2 - b^2$

$Ax = \lambda x$, $A^* x^* = \lambda^* x^*$

$\langle Ax, A^* x^* \rangle = \langle \lambda x, \lambda^* x^* \rangle = \lambda \lambda^* \langle x, x^* \rangle$

$\shortparallel$

$(A^* x^*)^T (Ax) = x^{*T} A^{*T} A x = x^{*T} A^2 x = \lambda^2 \langle x^*, x \rangle$

because A is symmetric and real !

$\Rightarrow \lambda = \lambda^*$

So that we can generalize this step!

For A is Complex, "Symmetric" means $A^* = A^T$