Linear Algebra

Midterm Sample Question

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Exercise True or False? In both cases, explain clearly.

- Every Diagnoal matrix is an invertible matrix. No
- An upper traingular matrix times an upper traingular matrix is a upper traingular matrix. Yes
- The inverse of a permutation matrix is also a permutation matrix. Yes
- The transpose of an ellimination matrix is also a an ellimination matrix. Yes
- If A and B are elimination matrix, then $AB = BA$. No
- Only symmetric matrix have a LDL decomposition. Yes
- The LU decomposition of a matrix is unique No
- The inverse of upper traingular matrix is lower traingular matrix No
- An Elimination matrix times an Elimination matrix matrix is stil an Elimination matrix. No
- Every invertible matrix is a square matrix. Yes
- $E_{21}E_{32}A$ means change Row 2 of matrix A by linear combination of Row 2 and Row 1 and then change Row 3 by linear combination of Row 3 and Row 2. No
- $\left\{ \begin{bmatrix} x \\ x+2y \end{bmatrix} | 3x+2y=0 \right\}$ is a vector space. Yes
- $\left\{ \begin{bmatrix} x \\ x+2y+1 \end{bmatrix} | 3x+2y=0 \right\}$ is a vector space. No
- All 3 by 3 matrices with the column vector $(1, 1, 1)$ in their column space forms a vector space. No

- The set of all polynomials of degree less than 3 forms a vector space. This means any polynomial that can be written in the form $ax^2 + bx + c$, where a, b and c are constants (which can include zero), belongs to this vector space. Yes

- A is an invertible matrix then $A^{-1}A^{\top}A^2$ is also invertible. Yes, the inverse matrix is $A^{-2}(A^{-1})^{-\top}A$

- For a matrix $A \in \mathbb{R}^{4\times5}$, the largest possible rank of A is 5. No

- For a matrix $A \in \mathbb{R}^{4\times5}$ there are possibility that linear system $Ax = b$ have one and only have one solution. No

- For a matrix $A \in \mathbb{R}^{4\times5}$, $\text{rank}(A) = 4$. There are possibility that linear system $Ax = b$ have one and only have one solution. No

- For a matrix $A \in \mathbb{R}^{4\times3}$, $\text{rank}(A) = 3$. There are possibility that linear system $Ax = b$ have one and only have one solution. Yes

- For a matrix $A \in \mathbb{R}^{5\times4}$, $\text{rank}(A) = 4$. There are possibility that linear system $Ax = b$ have one and only have one solution. Yes

- For a matrix $A \in \mathbb{R}^{4\times5}$, $\text{rank}(A) = 4$. There are possibility that linear system $Ax = b$ have no solution. No

- For a matrix $A \in \mathbb{R}^{5\times4}$, $\text{rank}(A) = 4$. There are possibility that linear system $Ax = b$ have no solution. Yes

- The exist matrixs A and matrix B, $\text{rank}(A) = 3$ and $\text{rank}(AB) = 4$. No

- $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}$ is a basis of $\mathbb{R}^3$. No

- The column vectors of a full column rank $m \times n$ matrix is a basis of $\mathbb{R}^m$. No

- The column vectors of a full row rank $m \times n$ matrix is a basis of $\mathbb{R}^m$. No

- The complete solution of linear system $Ax = b$ is $\vec{x} = \begin{bmatrix} 0 \\ 2 \\ 3 \\ 1 \\ 2 \end{bmatrix} + x_1 \begin{bmatrix} 1 \\ 0 \\ 2 \\ -1 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$, then $\dim(\text{col}(A)) = 3$ Yes

- All symmetric matrix $A \in \mathbb{R}^{3\times3}$ forms a vector space whose dimension is 6. Yes

- Suppose the matrices A and B have the same column space, then A and B have the same nullspaces. No
- Matrices A and its row echelon form always have the same column space. No
- If two $m \times n$ matrices A and B have the same 4 fundamental spaces, then $A = B$. No
- Suppose A and B have the same column space, then A and B have the same rank. Yes
- $\text{rank}(A) = \text{rank}(A^\top)$ Yes
- There exist a matrix A whose column space is spanned by $(1, 2, 3)$ and $(1, 0, 1)$and whose nullspace is spanned by $(1, 2, 3, 6)$ No, The dimensions of such a matrix must be 3 by 4 ($m = 3$ and $n = 4$). The dimension of the column space is 2, because the given vectors are independent. That means the dimension of the nullspace must be $4 - 2 = 2$. The null space cannot be spanned by 1 vector.
- $Y = AX$ and A is an invertible matrix, then $\text{rank}(Y) = \text{rank}(X)$. Yes, because $Y = AX$ so $\text{rank}(Y) \leq \text{rank}(X)$. For A is invertible matrix, so $X = A^{-1}Y$ which tells us $\text{rank}(X) \leq \text{rank}(Y)$. The only possiblity is $\text{rank}(Y) = \text{rank}(X)$

Questions

- Compute Angle, matrix product, inverse matrix, LU decomposition, LDU decomposition, complete solution
- Compute the rank of the four subspaces