Linear Algebra Cheat Sheet
Inverse Matrix

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1 Properties of Inverse Matrix

- $AA^{-1} = I$, $A^{-1}A = I$
- $(A^T)^{-1} = (A^{-1})^T$
- $(AB)^{-1} = B^{-1}A^{-1}$

Question 1 If A and M have inverse matrix A^{-1} and M^{-1} and

- $AX = B$
- $YM = C$
- $AZM^\top = D$

what is X, Y, Z?

2 Elimination

Elimination as Matrix Operation We can write the operations to change equivalent linear system by $[A|b] \to [E_{ij}A|E_{ij}b]$ and $[P_{ij}A|P_{ij}b]$.

- Elimination matrix E_{ij}:
    - Replace row (i) by $*$*row(i) + row (j)
    - Identity matrix except $a_{ij} = *$
- Permutation matrix P_{ij}:
    - Swtich Row (i) with Row (j)
    - Identity matrix except $a_{ij} = a_{ji} = 1, a_{ii} = a_{jj} = 0$

Question 2 What is the matrix after the following operations

- Change Row 2 of A to Row 2 + 2* Row 1
- Switch Row 3 and Row 4 of the new matrix
- Change Row 4 of the new matrix to Row 4 + 2* Row 2

3   Inverse Matrix

- The inverse of a matrix exists if and only if the matrix is a square matrix and all column vectors are linear independent.

- The inverse of a matrix exists if and only if elimination produced n non-zero pivots.

Questions (answer is in the slide) Can you describe how the upper triangular form and their pivots look like for the following three cases

- The linear system have a single solution

- The linear system have no solution

- The linear system have infinite solutions

Questions Please ensure you know the answer of the following questions

- How to calculate the inverse of a matrix?

- What is the inverse of the elimination matrix? What is the inverse of the permutation matrix?