Example

$$A = \begin{array}{c} \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{pmatrix} 4 & 5 & 6 \end{pmatrix} \end{array} \in \mathbb{R}^{2\times 3}$$

2 row # 3 column.

$m=2$
$n=3$

Column Representation. $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$

$\mathbb{R}^2$

$\vec{v_1} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$ $\vec{v_2} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ $\vec{v_3} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}$ $A = [\vec{v_1} \ \vec{v_2} \ \vec{v_3}]$

Row Representation $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$

$\mathbb{R}^3$

$\vec{r_1} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ $\vec{r_2} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$ $r_1^T = (1 \ 2 \ 3)$ $r_2^T = (4 \ 5 \ 6)$ $A = \begin{bmatrix} r_1^T \\ r_2^T \end{bmatrix}$

Matrix Vector Multiplication A can only Multiply a $\mathbb{R}^3$ vector

$$Ax = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + 2x_3 + 3x_3 \\ 4x_1 + 5x_1 + 6x_3 \end{pmatrix}$$

$\leftarrow (x_1 \ x_2 \ x_3) \cdot (1 \ 2 \ 3)$
$\leftarrow (x_1 \ x_2 \ x_3) \cdot (4 \ 5 \ 6)$ ← row Representa.

$$= x_1 \begin{pmatrix} 1 \\ 4 \end{pmatrix} + x_2 \begin{pmatrix} 2 \\ 5 \end{pmatrix} + x_3 \begin{pmatrix} 3 \\ 6 \end{pmatrix}$$ ← Column Representation.

Linear System $m=2$ Equations $n=3$ unknown Variables

$A\vec{x} = \vec{b}$ $\vec{b} \in \mathbb{R}^2$ $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ ← unknow Variables.

$$\hookrightarrow \begin{cases} 1x_1 + 2x_2 + 3x_3 = b_1 \\ 4x_1 + 5x_2 + 6x_3 = b_2 \end{cases}$$

if A have an inverse, the solution is $x = A^{-1}b$

not every matrix A have an inverse.

Abstract Exercise.

Ax $A \in \mathbb{R}^{m\times n}$ ← # m row # n column Matrix

$x \in \mathbb{R}^n$ ← n dimensional Vector.

$b \in \mathbb{R}^m$

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

all a_{ij} are scalar $a_{ij} \in \mathbb{R}$

Row vector: $\vec{r_1} = \begin{pmatrix} a_{11} \\ a_{12} \\ \vdots \\ a_{1n} \end{pmatrix}$ $\vec{r_2} = \begin{pmatrix} a_{21} \\ a_{22} \\ \vdots \\ a_{2n} \end{pmatrix}$ $\ldots$ $\vec{r_m} = \begin{pmatrix} a_{m1} \\ \vdots \\ a_{mn} \end{pmatrix} \in \mathbb{R}^n$ vectors $A = \begin{bmatrix} r_1^T \\ \vdots \\ r_m^T \end{bmatrix}$

Column vector: $\vec{v_1} = \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix}$ $\vec{v_2} = \begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix}$ $\ldots$ $\vec{v_n} = \begin{pmatrix} a_{1n} \\ \vdots \\ a_{mn} \end{pmatrix}$ vectors $\in \mathbb{R}^m$ $A = [\vec{v_1} \cdots \vec{v_n}]$

Linear System

$Ax = b$ $\quad\quad \vec{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \quad\quad \vec{b} = \begin{pmatrix} b_1 \\ \vdots \\ b_m \end{pmatrix}$

$A \in \mathbb{R}^{m \times n}$ $\quad$ # m Equation $\quad$ # n unknown Variables

$$\hookrightarrow \begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \cdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_n \end{cases}$$

$$\hookrightarrow A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & & a_{2n} \\ \vdots & & & \vdots \\ a_{m1} & a_{m2} & & a_{mn} \end{pmatrix}$$

x_i are scalar

$\vec{x}$ are Vector in $\mathbb{R}^n$

b_i are scalar

$\vec{b}$ are vector in $\mathbb{R}^m$

a_{ij} are scalar

A are $\mathbb{R}^{m \times n}$ matrix

$\mathbb{R}$: Scalar

$\mathbb{R}^n$: vector ($\mathbb{R}^{n \times 1}$ n by 1 matrix)

$\mathbb{R}^{m \times n}$ m by n Matrix