NYU

Lecture 2

Vectors and Spans

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Based on Dr. Ralph Chikhany's Slide

Reminders

- Get access to Gradescope, Campuswire.
- Obtain the textbook.
- Problem Set 1 due by 11.59 pm on Friday (NY time). 2 Weeks from Now
  - ✓ Late work policy applies.
- Recap Quiz 1 due by 11.59 pm on Sunday (NY time). Next Week
  - ❖ Late work policy does not apply.
- Recap Quiz is timed.
  - ❑ Once you start, you have 60 minutes to finish it (even if you close the tab)

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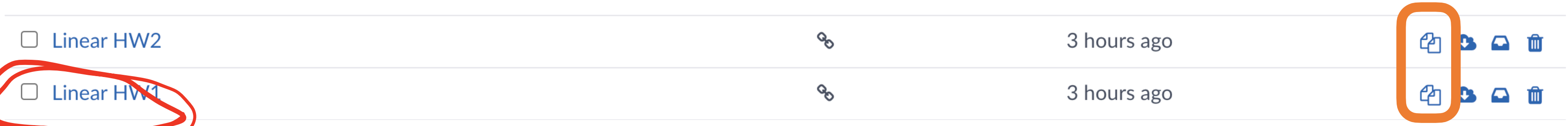


Copy

You can put what you want to recap in the (anonymous) form.

ReCap

Course notes adapted from *Introduction to Linear Algebra* by Strang (5th ed), N. Hammoud's NYU lecture notes, and *Interactive Linear Algebra* by Margalit and Rabinoff, in addition to our text

Vector Addition

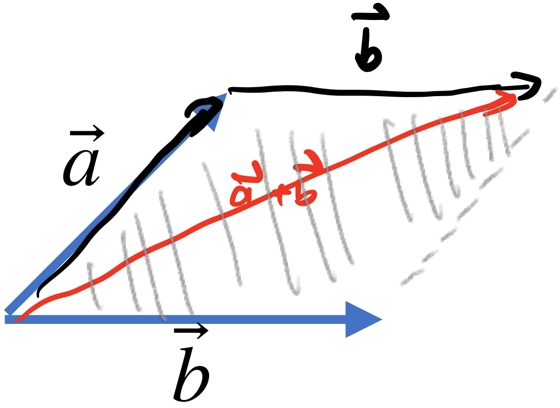
$$\begin{pmatrix}a\\b\\c\end{pmatrix}+\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}a+x\\b+y\\c+z\end{pmatrix}$$

- Can I add $\mathbb{R}^2$ vector with $\mathbb{R}^3$ vector? NO

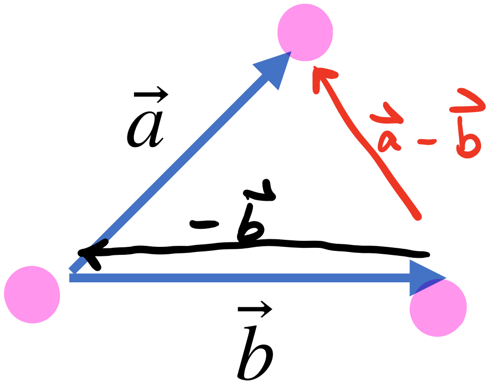
$\mathbb{R}^n$ can only add with $\mathbb{R}^n$

× add with $\mathbb{R}^m$ if $n \neq m$.

parallelogram Rule:



$\vec{a}+\vec{b}=?$ →



$\vec{a}-\vec{b}=?$

$\vec{a}+(-\vec{b})$

$\vec{a}-\vec{a}=?$ $\vec{0}$

$\vec{0}$ is not 0

Scalar vector multiplication

$$c\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} c \cdot x \\ c \cdot y \\ c \cdot z \end{pmatrix}$$

just change the length, but the same direction.

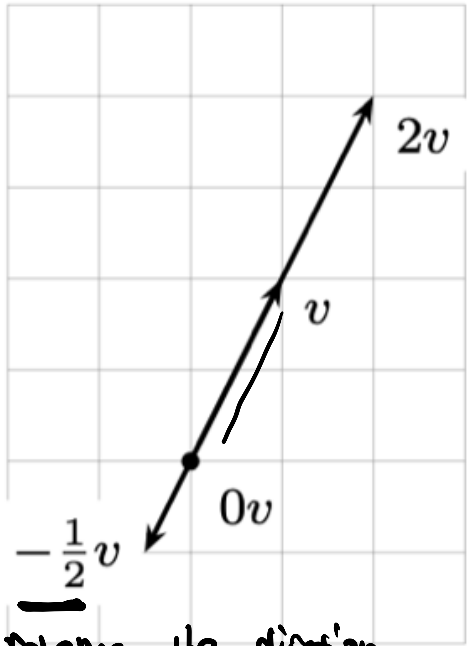
$0 \cdot \vec{v} = \vec{0}$ is not 0

$c(\vec{a} + \vec{b}) = c\vec{a} + c\vec{b}$ (v)

$$c\left(\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix}\right) = c\begin{pmatrix} x+a \\ y+b \end{pmatrix} = \begin{pmatrix} c(x+a) \\ c(y+b) \end{pmatrix}$$

$$= \begin{pmatrix} cx \\ cy \end{pmatrix} + \begin{pmatrix} ca \\ cb \end{pmatrix}$$

Some multiples of v.



reverse the direction.

Dot Product

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \bullet \begin{pmatrix} x \\ y \\ z \end{pmatrix} = ax + by + cz. \neq \begin{pmatrix} a \cdot x \\ b \cdot y \\ c z \end{pmatrix}$$

is scalar is not vector

weights or coef.

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = ax + by + cz$$

vector of coef.

it is linear combination of x. y. z

Dot product is a linear combination

"useful to understand like this in lecture"

Dot Product

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \bullet \begin{pmatrix} x \\ y \\ z \end{pmatrix} = ax + by + cz$$

Example 3 Dot products enter in economics and business. We have three goods to buy and sell. Their prices are (p_1, p_2, p_3) for each unit—this is the "price vector" $\boldsymbol{p}$. The quantities we buy or sell are (q_1, q_2, q_3)—positive when we sell, negative when we buy. *Selling* q_1 *units at the price* p_1 *brings in* $q_1 p_1$. The total income (quantities q times prices p) is ***the dot product*** $\boldsymbol{q} \cdot \boldsymbol{p}$ ***in three dimensions***:

$$\textbf{\textit{Income}} = \underbrace{(q_1, q_2, q_3)}^{\text{quantities}} \cdot \overbrace{(p_1, p_2, p_3)}^{\text{price}} = q_1 p_1 + q_2 p_2 + q_3 p_3 = \textit{\textbf{dot product}}.$$

Dot Product

$$\begin{pmatrix}a\\b\\c\end{pmatrix}\bullet\begin{pmatrix}x\\y\\z\end{pmatrix} = ax+by+cz$$

① if vector $\vec{v}$

$\|\vec{v}\|$: length.

② if scalar c

$|c|$: absolute value.

Length

$\underbrace{\|\vec{v}\|}_{\text{vector}} = \sqrt{\vec{v}\cdot\vec{v}}$

$\left\|\begin{pmatrix}a\\b\end{pmatrix}\right\| = \sqrt{a^2+b^2}$

$\|\overset{\text{scalar}}{c}\cdot\underbrace{\vec{v}}_{\text{vector}}\| = \underbrace{|c|}_{\text{scalar}}\cdot\|\vec{v}\|$.

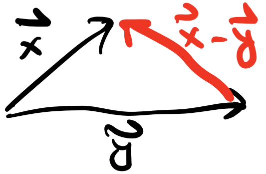
Ex $\underbrace{\|\vec{a}+\vec{b}\|}_{\text{Vector}}$

$\underbrace{|\vec{a}\cdot\vec{b}|}_{\text{Scalar}}$

$\|\vec{v}\|$ And $|c|$

Distance

dist $(x, y) = \|\vec{x}-\vec{y}\|$



Unit Vector: vector whose length is 1

$\dfrac{\vec{x}}{\|\vec{x}\|}$ is the unit the same direction as $\vec{x}$

What is the unit vector of (1,1)? with same direction

length of (1,1) is $\sqrt{2}$.

$$\frac{\begin{pmatrix}1\\1\end{pmatrix}}{\sqrt{2}} = \begin{pmatrix}1/\sqrt{2}\\1/\sqrt{2}\end{pmatrix}$$

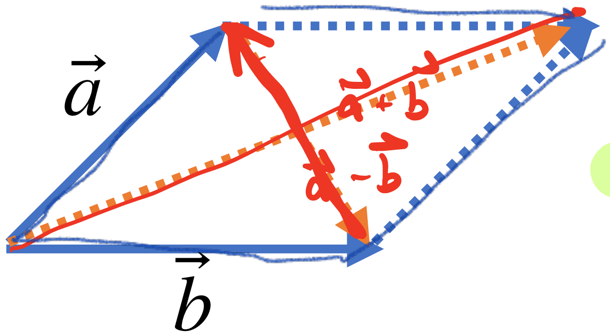
Dot Product

Communicative $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$

Distributive $(\vec{a}+\vec{b})\cdot\vec{c} = \vec{a}\cdot\vec{c} + \vec{b}\cdot\vec{c}$

L.C $(c_1\vec{a}+c_2\vec{b})\cdot\vec{c} = \underline{c_1}\,\vec{a}\cdot\vec{c} + \underline{c_2}\,\vec{b}\,\vec{c}$

Example $\|\vec{a}+\vec{b}\|^2 + \|\vec{a}-\vec{b}\|^2 = (\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}) + (\vec{a}-\vec{b})\cdot(\vec{a}-\vec{b})$



: $\vec{a}\cdot\vec{a} + \cancel{2\vec{a}\cdot\vec{b}} + \vec{b}\cdot\vec{b}$

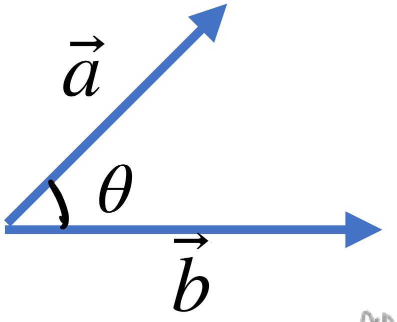
: $\vec{a}\cdot\vec{a} \;\cancel{-2\vec{a}\cdot\vec{b}} + \vec{b}\cdot\vec{b}$

$\Downarrow$ $\quad$ $\Downarrow$

$2\|\vec{a}\|^2$ $\quad$ $2\|\vec{b}\|^2$

$\Delta\quad \|\vec{a}+\vec{b}\|^2 + \|\vec{a}-\vec{b}\|^2 = 2\cdot\|\vec{a}\|^2 + 2\cdot\|\vec{b}\|^2$

Angle

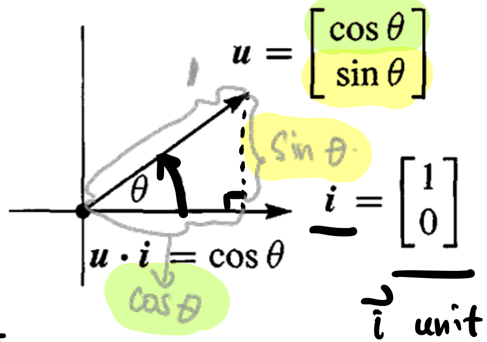


① Orthogonal $\vec{a} \perp \vec{b} = 0$

$\Leftrightarrow \|\vec{a} + \vec{b}\|^2 = \|\vec{a}\|^2 + \|\vec{b}\|^2 \Leftrightarrow \vec{a} \cdot \vec{b} = 0$

② $\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|\cdot\|\vec{b}\|} = \dfrac{(c\vec{a})\cdot\vec{b}}{\|c\vec{a}\|\cdot\|\vec{b}\|} = \dfrac{\cancel{c}(\vec{a}\cdot\vec{b})}{\cancel{|c|}\|\vec{a}\|\cdot\|\vec{b}\|}$

angle between $\vec{a}$ and $\vec{b}$ = ... $c\cdot\vec{a}$ and $\vec{b}$ $(c>0)$



$\vec{i}$ unit

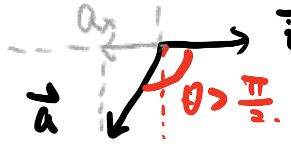
$\vec{u}\cdot\vec{i} = \begin{bmatrix}\cos\theta \\ \sin\theta\end{bmatrix}\begin{bmatrix}1 \\ 0\end{bmatrix}$ to x-axis

$= \cos\theta$

How to decide whether θ is larger than $\dfrac{\pi}{2}$

$\theta > \frac{\pi}{2} \Leftrightarrow \cos\theta < 0 \Leftrightarrow \vec{a}\cdot\vec{b} < 0$

the x-axis of $\vec{a}$ is negative



$\vec{i} = \begin{pmatrix}1 \\ 0\end{pmatrix}$

$\vec{a} = \begin{pmatrix}a_x \\ a_y\end{pmatrix}$ $\vec{a}\cdot\vec{i} = \begin{pmatrix}a_x \\ a_y\end{pmatrix}\cdot\begin{pmatrix}1 \\ 0\end{pmatrix} = a_x < 0$

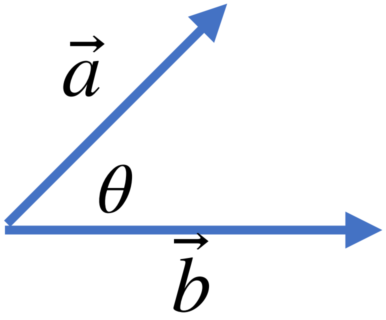
What is the unit vector parallel/orthogonal to $(4,3)$? Harder Question

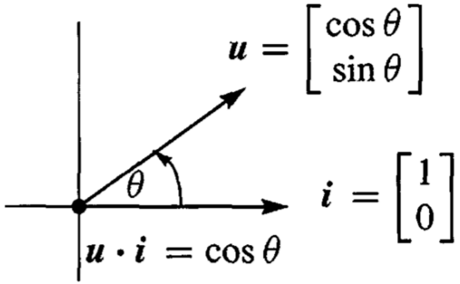
$\vec{x} \to \dfrac{\vec{x}}{\|\vec{x}\|}$. $\left\|\begin{pmatrix}4 \\ 3\end{pmatrix}\right\| = 5 \Rightarrow \begin{pmatrix}4/5 \\ 3/5\end{pmatrix}$

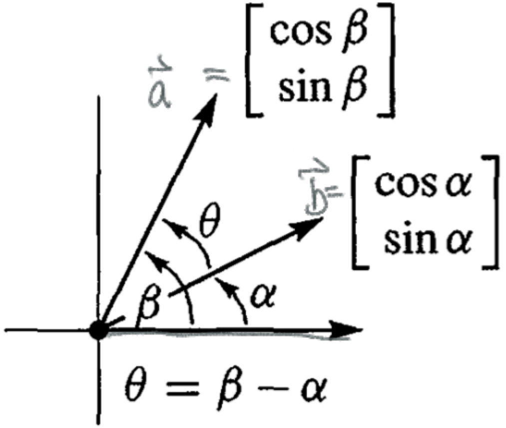
Harder ? orthogonal one. $\begin{pmatrix}-3/5 \\ 4/5\end{pmatrix}$ check $\begin{pmatrix}-3/5 \\ 4/5\end{pmatrix}\cdot\begin{pmatrix}4 \\ 3\end{pmatrix} = -3/5\times 4 + 4/5\times 3 = -12/5 + 12/5 = 0$

why? in later of lectures

Angle







calculate $\cos(\beta - \alpha)$

the angle between $\vec{a} \cdot \vec{b}$ is $\theta = \beta - \alpha$

$$\cos(\beta - \alpha) = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \cdot \|\vec{b}\|} = \frac{\begin{bmatrix}\cos\beta \\ \sin\beta\end{bmatrix} \begin{bmatrix}\cos\alpha \\ \sin\alpha\end{bmatrix}}{\underbrace{\|\vec{a}\|}_{\text{unit vec} \,=\, 1} \cdot \underbrace{\|\vec{b}\|}_{\text{unit vector} \,=\, 1}} = \cos\beta \cdot \cos\alpha + \sin\beta \cdot \sin\alpha$$

Motivation: Best fit of linear equation

Not Required

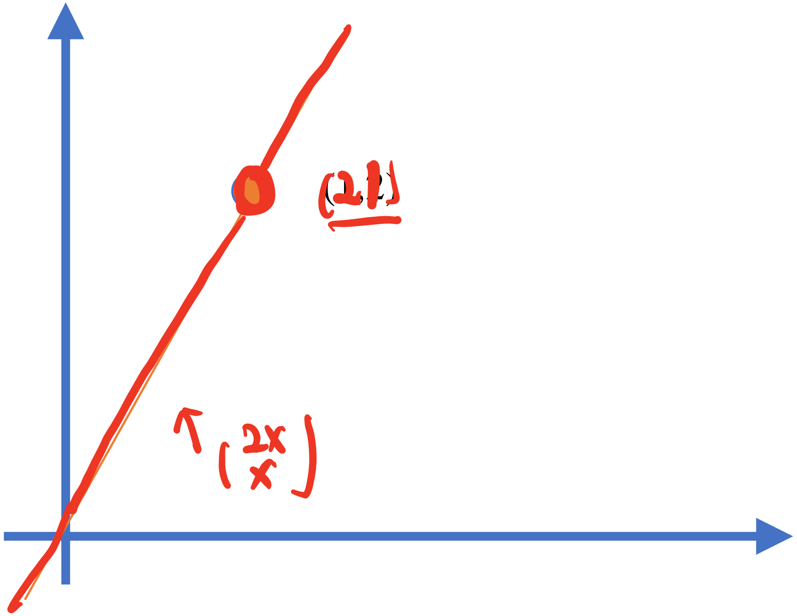
overdetermined linear system

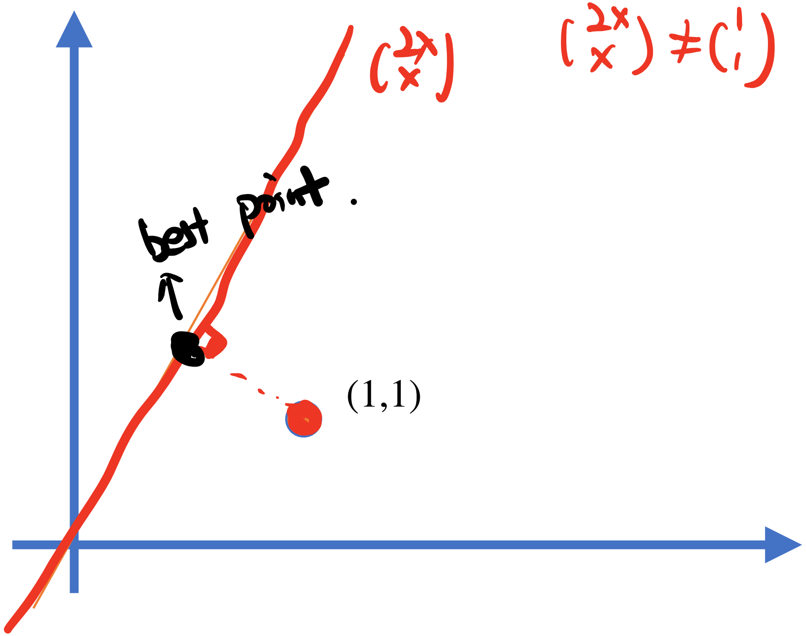
$x=1$

$$2x = 2$$
$$x = 1 \Rightarrow \underline{\begin{pmatrix} 2x \\ x \end{pmatrix}} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$2x = 1$$
$$x = 1$$

$\begin{pmatrix} 2x \\ x \end{pmatrix} \neq \begin{pmatrix} 1 \\ 1 \end{pmatrix}$





Example

1.2 C Find a vector $\boldsymbol{x} = (c, d)$ that has dot products $\boldsymbol{x} \cdot \boldsymbol{r} = 1$ and $\boldsymbol{x} \cdot \boldsymbol{s} = 0$ with the given vectors $\boldsymbol{r} = (2, -1)$ and $\boldsymbol{s} = (-1, 2)$.

How is this question related to Example **1.1 C**, which solved $c\boldsymbol{v} + d\boldsymbol{w} = \boldsymbol{b} = (1, 0)$?

$$\vec{x} \cdot \vec{r} = \begin{pmatrix} c \\ d \end{pmatrix} \begin{pmatrix} 2 \\ -1 \end{pmatrix} = 2c - d = 1$$

$$\vec{x} \cdot \vec{s} = \begin{pmatrix} c \\ d \end{pmatrix} \begin{pmatrix} -1 \\ 2 \end{pmatrix} = -c + 2d = 0$$

$$\longrightarrow \quad c \cdot \begin{bmatrix} 2 \\ -1 \end{bmatrix} + d \cdot \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

L.C. / Linear System.

by lectures 3. Matrices.

the same.

1.1 C Find two equations for the unknowns c and d so that the linear combination $c\boldsymbol{v} + d\boldsymbol{w}$ equals the vector $\boldsymbol{b}$:

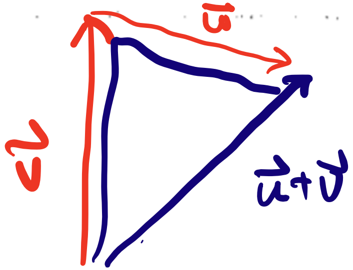
$$\boldsymbol{v} = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \qquad \boldsymbol{w} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \qquad \boldsymbol{b} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

Inequalities

SCHWARZ INEQUALITY $\quad |\boldsymbol{v} \cdot \boldsymbol{w}| \leq \|\boldsymbol{v}\| \, \|\boldsymbol{w}\|$

$|\cos\theta| < 1 \Rightarrow \dfrac{|v \cdot w|}{\|v\| \cdot \|w\|} < 1$

TRIANGLE INEQUALITY $\quad \|\boldsymbol{v} + \boldsymbol{w}\| \leq \|\boldsymbol{v}\| + \|\boldsymbol{w}\|$



$\|\vec{u}+\vec{v}\| \leq \|\vec{u}\| + \|\vec{v}\|$

Example 6 The dot product of $\boldsymbol{v} = (a, b)$ and $\boldsymbol{w} = (b, a)$ is $2ab$. Both lengths are $\sqrt{a^2 + b^2}$. The Schwarz inequality in this case says that $2ab \leq a^2 + b^2$.

Reminder: Linear Combination

$$\vec{w} = c_1\vec{v}_1 + c_2\vec{v}_2 + \cdots + c_p\vec{v}_p$$

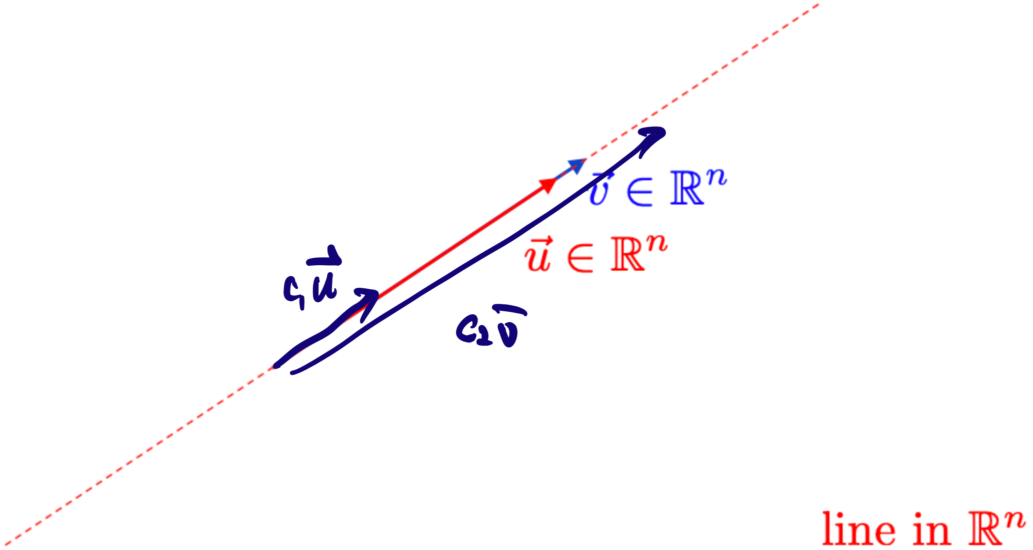
where $c_1, c_2, \ldots, c_p$ are scalars, $v_1, v_2, \ldots, v_p$ are vectors in $\mathbf{R}^n$, and w is a vector in $\mathbf{R}^n$.

Definition

We call w a **linear combination** of the vectors $v_1, v_2, \ldots, v_p$. The scalars $c_1, c_2, \ldots, c_p$ are called the **weights** or **coefficients**.

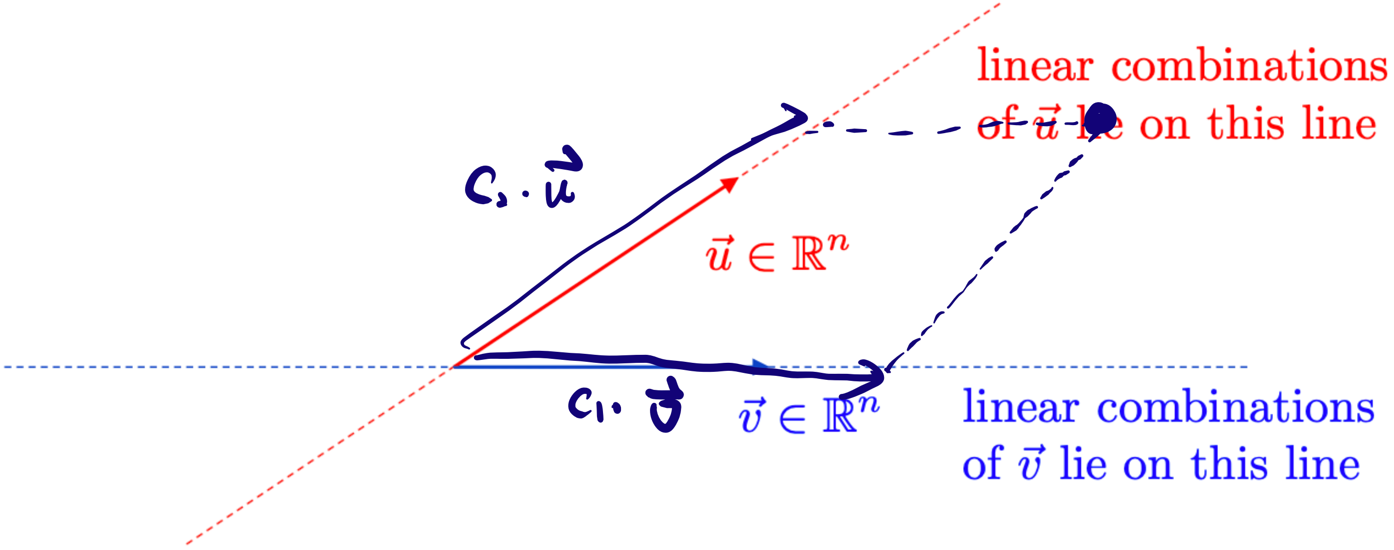
Geometric Interpretation of Linear Combinations

$\vec{u} \parallel \vec{v} \Rightarrow$ L.C. will form a line in $\mathbb{R}^n$



Geometric Interpretation of Linear Combinations

if $\vec{u} \nparallel \vec{v}$ L.C. forms the whole $\mathbb{R}^2$



linear combinations of $\vec{u}$ and $\vec{v}$ lie on a plane in $\mathbb{R}^2$

Transfer Linear Equation to a Linear Combination Problem

$$\begin{cases} 2x + y = 1 \\ x + y = 1 \end{cases} \Leftrightarrow x \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix} + y \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

the same

$\Leftrightarrow$ is $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ be represented by L.C. of vector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$?

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} = x \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix} + y \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2x + y \\ x + y \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = x \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} + y \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 2x+y \\ x+y \\ 2x+3y \end{pmatrix} \Rightarrow \begin{cases} 2x + y = 1 \\ x + y = 1 \\ 2x + 3y = 0 \end{cases}$$

Spans

Course notes adapted from *Introduction to Linear Algebra* by Strang (5th ed), N. Hammoud's NYU lecture notes, and *Interactive Linear Algebra* by Margalit and Rabinoff, in addition to our text

Reminder: Linear Combination

$$w = c_1 v_1 + c_2 v_2 + \cdots + c_p v_p$$

where $c_1, c_2, \ldots, c_p$ are scalars, $v_1, v_2, \ldots, v_p$ are vectors in $\mathbf{R}^n$, and w is a vector in $\mathbf{R}^n$.

Definition

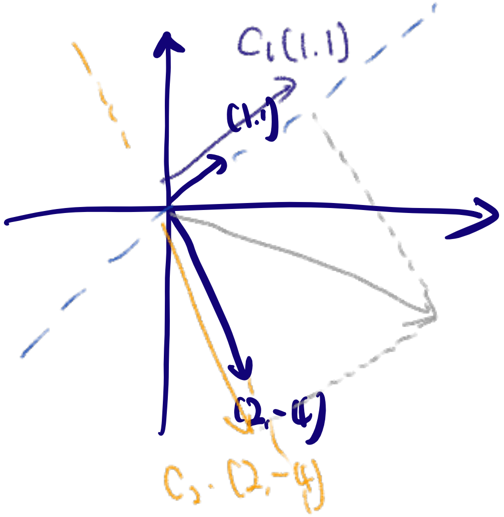
We call w a **linear combination** of the vectors $v_1, v_2, \ldots, v_p$. The scalars $c_1, c_2, \ldots, c_p$ are called the **weights** or **coefficients**.

Span

Let $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_m$ be a set of vectors in $\mathbb{R}^n$. We define

$$\text{span}\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_m\} = \text{set of all linear combinations of } \vec{v}_1, \vec{v}_2, \ldots, \vec{v}_m$$

For example, what is the span of $(2, -4)$ and $(1, 1)$?



① Answer: $\mathbb{R}^2$ is the span.

Span

Let $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_m$ be a set of vectors in $\mathbb{R}^n$. We define

$$\text{span}\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_m\} = \text{set of all linear combinations of } \vec{v}_1, \vec{v}_2, \ldots, \vec{v}_m$$

Is $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ in the span of $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$?

Algebra: is there a scalar c

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} = c \cdot \begin{bmatrix} 2 \\ 4 \end{bmatrix} \text{ ?} \qquad \text{Yes, } \quad c = 1/2$$

Is $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ in the span of $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$?

$$\begin{bmatrix} 1 \\ -2 \end{bmatrix} = c \cdot \begin{bmatrix} 2 \\ 4 \end{bmatrix} \text{ ?} \qquad \text{No}$$

$\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ is not in span.

Span

Let $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_m$ be a set of vectors in $\mathbb{R}^n$. We define

$$\text{span}\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_m\} = \text{set of all linear combinations of } \vec{v}_1, \vec{v}_2, \ldots, \vec{v}_m$$

Is $\begin{bmatrix} 4 \\ -2 \end{bmatrix}$ in the span of $\begin{bmatrix} 2 \\ -4 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$?

Is there c_1 and c_2 s.t. $\begin{bmatrix} 4 \\ -2 \end{bmatrix} = c_1 \begin{bmatrix} 2 \\ -4 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

in Lecture 4

Linear System. For Now Guess! $c_1 = 1$, $c_2 = 2$.

More Precise Definition

"the set of" "such that"

Definition

Let $v_1, v_2, \ldots, v_p$ be vectors in $\mathbf{R}^n$. The **span** of $v_1, v_2, \ldots, v_p$ is the collection of all linear combinations of $v_1, v_2, \ldots, v_p$, and is denoted $\text{Span}\{v_1, v_2, \ldots, v_p\}$. In symbols:

$$\boxed{\text{Span}\{v_1, v_2, \ldots, v_p\} = \left\{ x_1v_1 + x_2v_2 + \cdots + x_pv_p \;\middle|\; x_1, x_2, \ldots, x_p \text{ in } \mathbf{R} \right\}.}$$

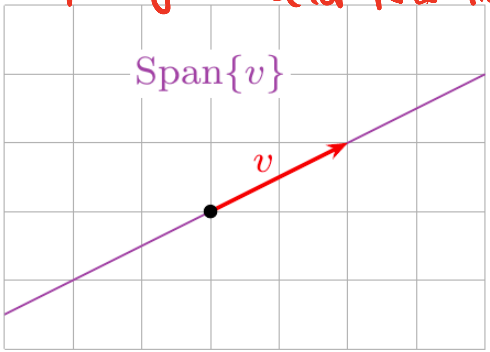
Synonyms: $\text{Span}\{v_1, v_2, \ldots, v_p\}$ is the subset **spanned by** or **generated by** $v_1, v_2, \ldots, v_p$.

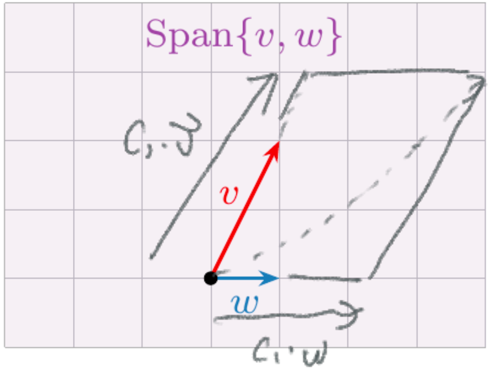
This is the first of several definitions in this class that you simply **must learn**. I will give you other ways to think about Span, and ways to draw pictures, but *this is the definition*. Having a vague idea what Span means will not help you solve any exam problems!

Span in $\mathbb{R}^2$

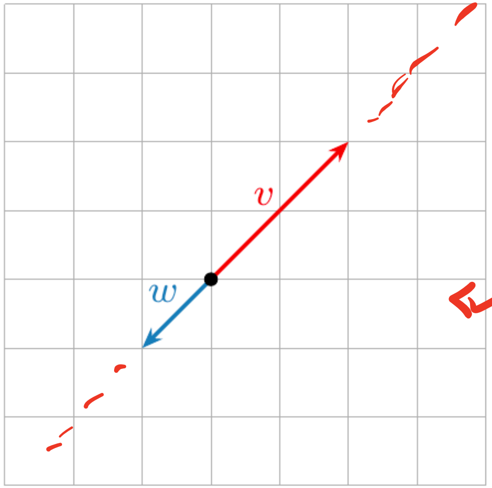
Drawing a picture of Span$\{v_1, v_2, \ldots, v_p\}$ is the same as drawing a picture of all linear combinations of $v_1, v_2, \ldots, v_p$.

span of a single vector is a line.





if $\vec{v}$ and $\vec{w}$ are not parallel then the span is whole $\mathbb{R}^2$.



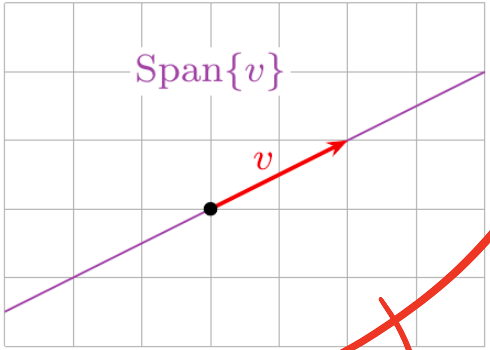
if $\vec{v}$ and $\vec{w}$ are parallel.

then the span is a line

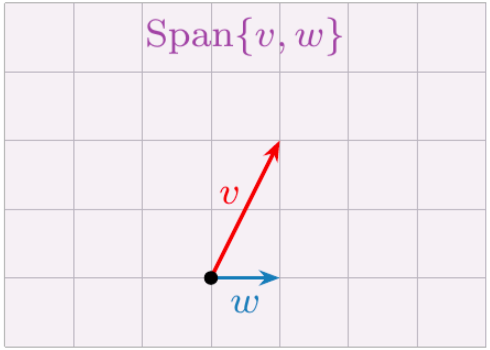
Span in $\mathbb{R}^2$

Drawing a picture of $\text{Span}\{v_1, v_2, \ldots, v_p\}$ is the same as drawing a picture of all linear combinations of $v_1, v_2, \ldots, v_p$.

1 vector

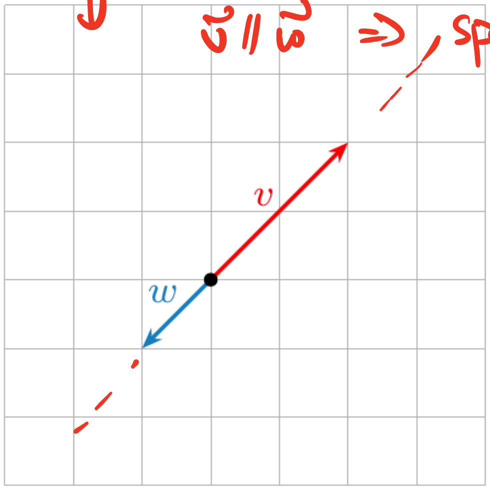


$\vec{w} \nparallel \vec{v}$



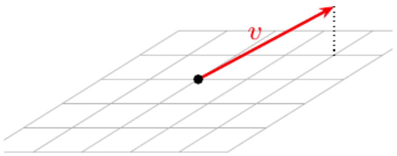
$\vec{w} \notin \text{span}\{\vec{v}\}$

$\vec{v} \parallel \vec{w} \Rightarrow$ span is a line

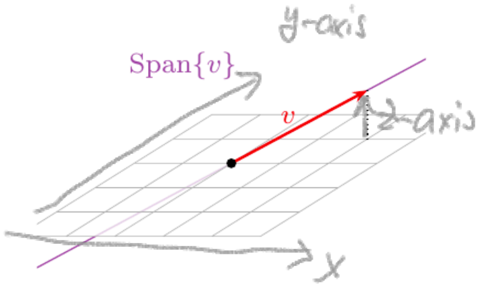


$\vec{v} \in \text{span}\{w\}$

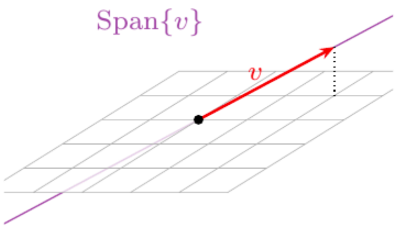
Span in $\mathbb{R}^3$

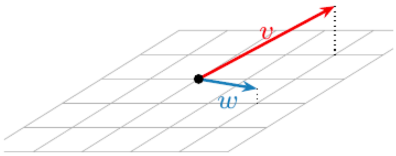


Span in $\mathbb{R}^3$



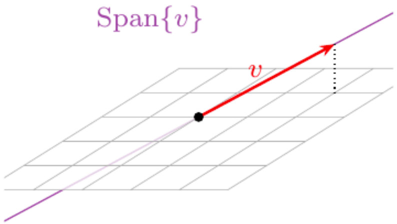
Span in $\mathbb{R}^3$

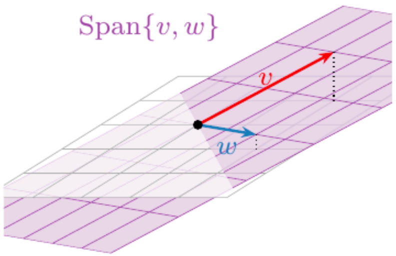




$\mathbb{R}^2 \neq \mathbb{R}^3$

Span in $\mathbb{R}^3$

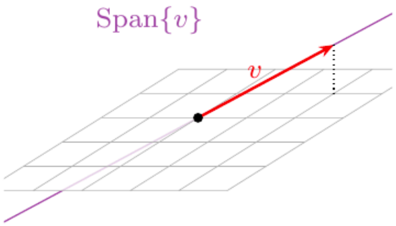


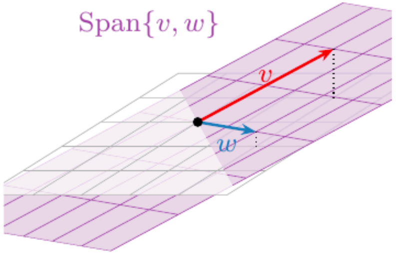


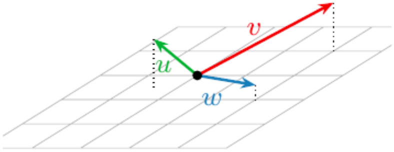
purple plane is the span

isn't $\mathbb{R}^3$

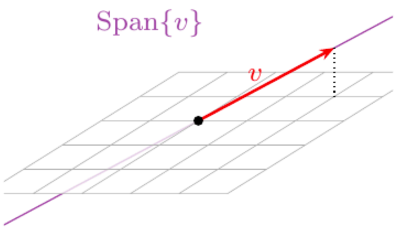
Span in $\mathbb{R}^3$

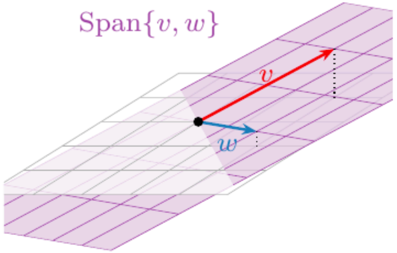


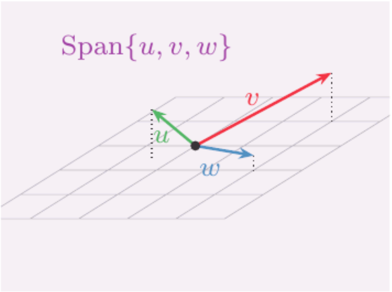




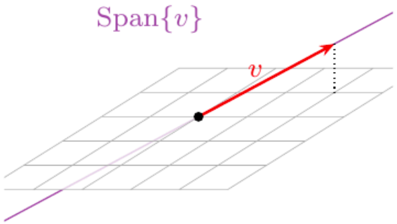
Span in $\mathbb{R}^3$

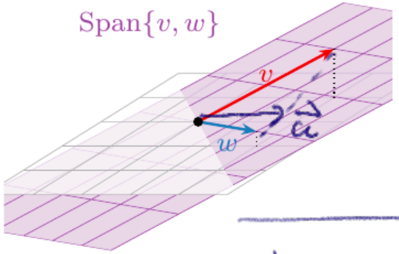




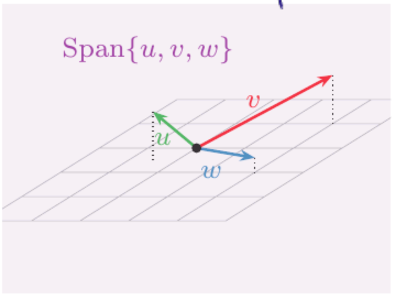


Span in $\mathbb{R}^3$





Case 1 $\vec{u}$ is not in the plane

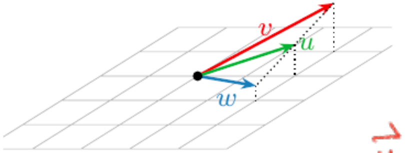


is not in

$\vec{u} \notin \text{span}\{v, w\}$

then $\text{span}\{\vec{u}, \vec{v}, \vec{w}\}$ is the $\mathbb{R}^3$

Case 2 $\vec{u}$ lies on plane, the span will not change.



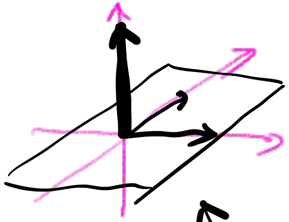
lies in

$\vec{u} \in \text{span}\{v, w\}$

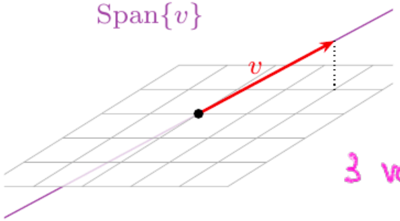
if $\vec{u}$ is in the span $\{v, w\}$

$\text{span}\{u, v, w\}$ is the same as $\text{span}\{v, w\}$

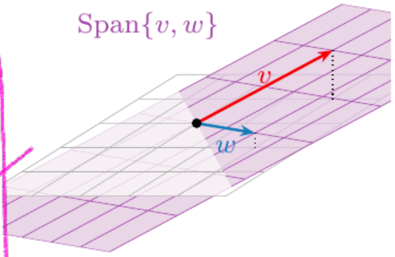
Span in $\mathbb{R}^3$



1 vector

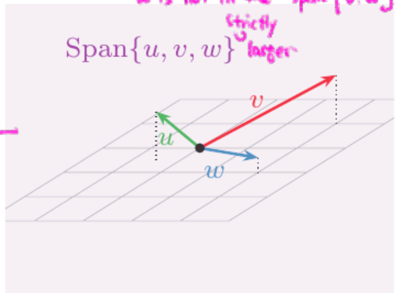


2 vector



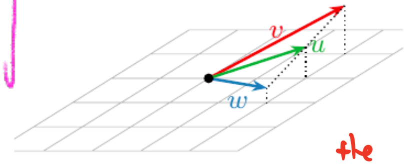
3 vector

$\vec{u}$ is not in the span $\{\vec{v}, \vec{w}\}$



strictly larger

whole $\mathbb{R}^3$ space



if the third vector $\vec{v}$ is a L.C. of $\vec{u}$, $\vec{w}$

$\vec{v}$ is not helpful

the span is still a plane.

Thm. $\vec{v} = a_1\vec{u} + a_2\vec{w}$

then $\text{span}\{\vec{v}, \vec{u}, \vec{w}\} = \text{span}\{\vec{u}, \vec{w}\}$

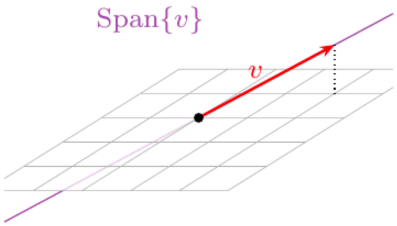
if $\vec{x} \in \text{span}\{\vec{v}, \vec{u}, \vec{w}\}$

$$\vec{x} = c_1\vec{u} + c_2\vec{w} + c_3\vec{v}$$
$$= c_1\vec{u} + c_2\vec{w} + c_3(a_1\vec{u} + a_2\vec{w})$$
$$= (c_1 + c_3 a_1)\vec{u} + (c_2 + c_3 a_2)\vec{w}$$

if $\vec{x} \in \text{span}\{\vec{u}, \vec{w}\}$

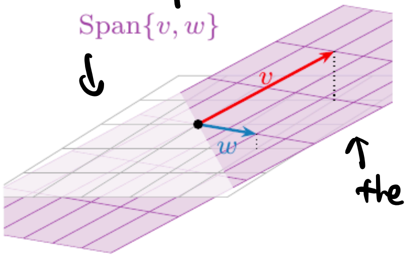
$$\vec{x} = c_1 \cdot \vec{u} + c_2 \cdot \vec{w} = \vec{0} + \dots = 0 \cdot \vec{v} + c_1 \cdot \vec{u} + c_2 \cdot \vec{w}$$
$$\Rightarrow \vec{x} \in \text{span}\{\vec{v}, \vec{u}, \vec{w}\}$$

Span in $\mathbb{R}^3$

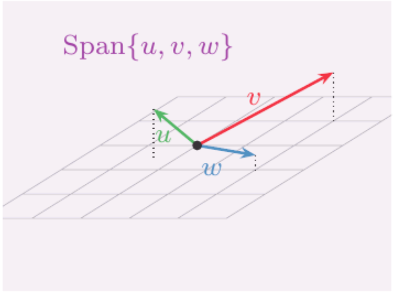


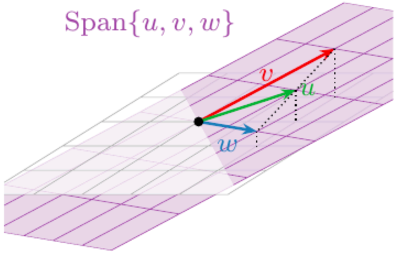
x-axis y-axis

white space is not the span



the purple part is the span





Questions?