Linear Algebra – Final Exam Review

1 True or False?

1. If $\vec{v_1}$ and $\vec{v_2}$ are linearly independent e-vectors of A, then they correspond to different e-values.
2. $\lambda \in \mathbb{R}$ is an eigenvalue of A if and only if there is a nonzero solution to $(A - \lambda I)\vec{x} = 0$.
3. To find the eigenvectors of A, we reduce the matrix A to row echelon form.
4. Any invertible matrix is diagonalizable.
5. Two matrices that have the same eigenvalues (with same multiplicities) should be similar.
6. If A (square matrix) is diagonalizable and invertible, then (choose the right answer)
   - Its inverse is diagonalizable and is invertible
   - Its inverse is diagonalizable and could be invertible
   - Its inverse could be diagonalizable and is invertible
   - Its inverse could be diagonalizable and could be invertible
   - We need more information to determine this

2 Some Proofs

1. Suppose a square matrix A is such that its square is zero. Find all eigenvalues of A.
2. We say that a matrix M is idempotent if $M^n = M$ for all positive integers n. Let A and B be two similar matrices. Show that if one of those matrices is idempotent, then the other is.
3. Let A be an $n \times n$ matrix with determinant 5 . What is the determinant of $-A$?
4. Let A be an orthogonal 2×2 matrix. Show that $\|A\vec{x}\| = \|\vec{x}\|$ for all vectors $\vec{x}$ in $\mathbb{R}^2$. Is this result generalizable (for any $n \times n$ matrix)?