

Properties of Determinate

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Four Basic Property

- (1) $\det([v_1, \dots, v_i, \dots, v_j, \dots, v_n]) = -\det([v_1, \dots, v_j, \dots, v_i, \dots, v_n])$ (switching i, j column make the determinate negative)
- (2) $\det(AB) = \det(A)\det(B)$
- (linear combination of single column)
 - (3) $\det([v_1+v'_1, v_2, \dots, v_n]) = \det([v_1, v_2, \dots, v_n]) + \det([v'_1, v_2, \dots, v_n])$
 - (4) $\det([cv_1, v_2, \dots, v_n]) = c\det([v_1, v_2, \dots, v_n])$

determinates of basic matrix

- $\det(I_n) = 1$
- What is the determinate of a diagonal matrix?
- What is the determinate of an orthogonal matrix matrix?
- What is the determinate of a permutation matrix?
- What is the determinate of an elimination matrix?
- What is the determinate of a lower diagonal matrix?

Try to prove the following property:

- $\det([\vec{0}, v_1, v_2, \dots, v_{n-1}]) = 0$
- $\det([v_1, v_1, v_2, \dots, v_{n-1}]) = 0$ (A have two equal columns, then determinate is 0)
- $\det(cA) = c^n \det(A)$
- $\det(A) = \det(A^T)$ (*hint*:LU Decomposition)