Properties of Determinate

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Four Basic Property

- (1) $\det([v_1, \cdots, v_i, \cdots, v_j, \cdots, v_n]) = -\det([v_1, \cdots, v_j, \cdots, v_i, \cdots, v_n])$ (switching i, j column make the determinate negative)
- (2) $\det(AB) = \det(A)\det(B)$
- (linear combination of single column)
  - (3) $\det([v_1 + v_1', v_2, \cdots, v_n]) = \det([v_1, v_2, \cdots, v_n]) + \det([v_1', v_2, \cdots, v_n])$
  - (4) $\det([cv_1, v_2, \cdots, v_n]) = c\det([v_1, v_2, \cdots, v_n])$

determinates of basic matrix

- $\det(I_n) = I$
- What is the determinate of a diagonal matrix?
- What is the determinate of an orthogonal matrix matrix?
- What is the determinate of a permutation matrix?
- What is the determinate of an elimination matrix?
- What is the determinate of a lower diagonal matrix?

Try to prove the following property:

- $\det([\vec{0}, v_1, v_2, \cdots, v_{n-1}]) = 0$
- $\det([v_1, v_1, v_2, \cdots, v_{n-1}]) = 0$ (A have two equal columns, then determiante is 0)
- $\det([c_1v_1, c_2v_2, \cdots, v_n]) = c_1c_2\det([v_1, v_2, \cdots, v_n])$
- $\det(cA) = c^n\det(A)$
- $\det(A) = \det(A^T)$ (*hint*:LU Decomposition)