Eigen Vectors.

$$A x_i = \lambda_i x_i$$

$$\Rightarrow A \underbrace{[x_1 \cdots x_n]}_{P} = [\lambda_1 x_1 \cdots \lambda_n x_n] = \underbrace{[x_1 \cdots x_n]}_{P} \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix}$$

$$\Rightarrow A = P \underset{\uparrow \text{ diagnol}}{\Sigma} P^{-1}$$

A Symmetric

$$P^{-T} = (P^{-1})^T = (P^T)^{-1}$$

$$A = P \Sigma P^{-1}$$

$$\Vert$$

$$A^T = P^{-T} \Sigma P^T \quad \Rightarrow P = P^{-T} \Leftrightarrow \underset{\Uparrow \text{ orthognol Matrix}}{P^T P = P P^T = I}$$

$$\Downarrow$$

$A = P \Sigma P^T$ if A symmetric P orthogonl.

SVD

$A = Q \Sigma P$ — Q: eigen of AA^T, P: eigen of A^TA. Q, P orthogonl. Σ diag

- A^TA is symmetric

$$A^TA = (Q \Sigma P)^T Q \Sigma P = P^T \Sigma \underbrace{Q^T Q}_{I} \Sigma P$$

$$= P^T \underbrace{\Sigma \Sigma}_{\text{diagnl.}} P$$

- AA^T is symmetric. $AA^T = Q^T \Sigma \Sigma Q$.

Orthogonal Matrix

$$P^T P = I$$

write down P using column Representation. $P = [v_1 \cdots v_n]$

$$P^T = \begin{bmatrix} v_1^T \\ \vdots \\ v_n^T \end{bmatrix} \quad \text{Then } P^T P = \begin{bmatrix} v_1^T v_1 & v_1^T v_2 & \cdots & v_1^T v_n \\ v_2^T v_1 & v_2^T v_2 & \cdots & v_2^T v_n \\ \vdots & & & \vdots \\ v_n^T v_1 & v_n^T v_2 & \cdots & v_n^T v_n \end{bmatrix}$$

$P^T P = I_n$ means.

$v_1^T v_1 = v_2^T v_2 = \cdots = v_n^T v_n = 1$

$v_i^T v_j = 0$ (if $i \neq j$)

means. v_i have no correlation with v_j

Projection using Orthogonal Basis. $A = [v_1 \cdots v_n]$

Projection: $\underbrace{(A^T A)^{-1}}_{I_n \text{ by orthogonal}} A^T b = \begin{bmatrix} v_1^T b \\ \vdots \\ v_n^T b \end{bmatrix}$

So $\underbrace{A}_{[v_1 \cdots v_n]} \underbrace{(A^T A)^{-1} A^T b}_{\begin{bmatrix} v_1^T b \\ v_n^T b \end{bmatrix}} = \underbrace{(v_1^T b)}_{\text{coef}} \underbrace{v_1}_{\text{basis}} + \cdots + (v_n^T b) v_n$

PCA. $X \leftarrow$ data

$X = \overbrace{[\,|\cdots\,|\,]}^{n \ \#\text{data}} \} p \ \#\text{feature}.$

SVD $X = Q \Sigma P$.

Σ: same size as x.

Q: #feature $\times$ # featue → principle. component. $\Rightarrow$ eigen of XX^T (covariane. matrix) are orthogonl basis

P: #data $\times$ #data.

Q. P are orthogonl.

- correlation between principle component are 0.
- Σ: feature importane $\Rightarrow$ eigen value of XX^T

$v_1 \cdots v_k$ are first k- principle component.

- $\underset{\text{i-th data}}{X_i} \simeq \underbrace{(X_i^T v_1) v_1 + (X_i^T v_2) v_2 + \cdots + (X_i^T v_k) v_k}_{\text{The best } k \text{ features to reconstruct the data.!}}$